Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding the Derivative by the Limit Process In Exercises 1–4, find the derivative of the function by the limit process.

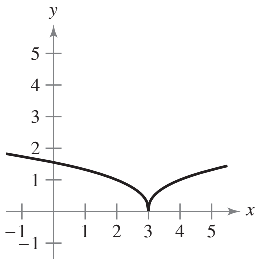
1. $f(x) = 12$
2. $f(x) = 5x - 4$
3. $f(x) = x^2 - 4x + 5$
4. $f(x) = \dfrac{6}{x}$

Using the Alternative Form of the Derivative In Exercises 5 and 6, use the alternative form of the derivative to find the derivative at $x = c$ (if it exists).

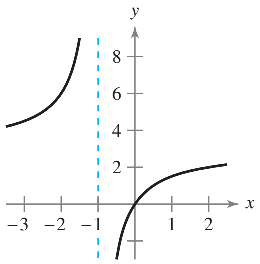
5. $g(x) = 2x^2 - 3x, \quad c = 2$
6. $f(x) = \dfrac{1}{x + 4}, \quad c = 3$

Determining Differentiability In Exercises 7 and 8, describe the x-values at which f is differentiable.

7. $f(x) = (x - 3)^{2/5}$



8. $f(x) = \dfrac{3x}{x + 1}$



Finding a Derivative In Exercises 9–20, use the rules of differentiation to find the derivative of the function.

9. $y = 25$
10. $f(t) = 4t^4$
11. $f(x) = x^3 - 11x^2$
12. $g(s) = 3s^5 - 2s^4$
13. $h(x) = 6\sqrt{x} + 3\sqrt[3]{x}$
14. $f(x) = x^{1/2} - x^{-1/2}$
15. $g(t) = \dfrac{2}{3t^2}$
16. $h(x) = \dfrac{8}{5x^4}$
17. $f(\theta) = 4\theta - 5 \sin \theta$
18. $g(\alpha) = 4 \cos \alpha + 6$
19. $f(\theta) = 3 \cos \theta - \dfrac{\sin \theta}{4}$
20. $g(\alpha) = \dfrac{5 \sin \alpha}{3} - 2\alpha$

Finding the Slope of a Graph In Exercises 21–24, find the slope of the graph of the functions at the given point.

21. $f(x) = \dfrac{27}{x^3}, \quad (3, 1)$
22. $f(x) = 3x^2 - 4x, \quad (1, -1)$
23. $f(x) = 2x^4 - 8, \quad (0, -8)$
24. $f(\theta) = 3 \cos \theta - 2\theta, \quad (0, 3)$

25. **Vibrating String** When a guitar string is plucked, it vibrates with a frequency of $F = 200\sqrt{T}$, where F is measured in vibrations per second and the tension T is measured in pounds. Find the rates of change of F when (a) $T = 4$ and (b) $T = 9$.

26. **Volume** The surface area of a cube with sides of length ℓ is given by $S = 6\ell^2$. Find the rates of change of the surface area with respect to ℓ when (a) $\ell = 3$ inches and (b) $\ell = 5$ inches.

Vertical Motion In Exercises 27 and 28, use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects.

27. A ball is thrown straight down from the top of a 600-foot building with an initial velocity of -30 feet per second.
    (a) Determine the position and velocity functions for the ball.
    (b) Determine the average velocity on the interval $[1, 3]$.
    (c) Find the instantaneous velocities when $t = 1$ and $t = 3$.
    (d) Find the time required for the ball to reach ground level.
    (e) Find the velocity of the ball at impact.

28. To estimate the height of a building, a weight is dropped from the top of the building into a pool at ground level. The splash is seen 9.2 seconds after the weight is dropped. What is the height (in feet) of the building?

Finding a Derivative In Exercises 29–40, use the Product Rule or the Quotient Rule to find the derivative of the function.

29. $f(x) = (5x^2 + 8)(x^2 - 4x - 6)$
30. $g(x) = (2x^3 + 5x)(3x - 4)$
31. $h(x) = \sqrt{x} \sin x$
32. $f(t) = 2t^5 \cos t$
33. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$
34. $f(x) = \dfrac{2x + 7}{x^2 + 4}$
35. $y = \dfrac{x^4}{\cos x}$
36. $y = \dfrac{\sin x}{x^4}$
37. $y = 3x^2 \sec x$
38. $y = 2x - x^2 \tan x$
39. $y = x \cos x - \sin x$
40. $g(x) = 3x \sin x + x^2 \cos x$

Finding an Equation of a Tangent Line In Exercises 41–44, find an equation of the tangent line to the graph of f at the given point.

41. $f(x) = (x + 2)(x^2 + 5), \quad (-1, 6)$
42. $f(x) = (x - 4)(x^2 + 6x - 1), \quad (0, 4)$
43. $f(x) = \dfrac{x + 1}{x - 1}, \quad \left(\dfrac{1}{2}, -3\right)$
44. $f(x) = \dfrac{1 + \cos x}{1 - \cos x}, \quad \left(\dfrac{\pi}{2}, 1\right)$

Finding a Second Derivative In Exercises 45–50, find the second derivative of the function.

45. $g(t) = -8t^3 - 5t + 12$
46. $h(x) = 6x^{-2} + 7x^2$
47. $f(x) = 15x^{5/2}$
48. $f(x) = 20\sqrt[5]{x}$
49. $f(\theta) = 3 \tan \theta$
50. $h(t) = 10 \cos t - 15 \sin t$

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51. Acceleration The velocity of an object in meters per second is $v(t) = 20 - t^2$, $0 \le t \le 6$. Find the velocity and acceleration of the object when $t = 3$.

52. Acceleration The velocity of an automobile starting from rest is

$$v(t) = \frac{90t}{4t + 10}$$

where v is measured in feet per second. Find the acceleration at (a) 1 second, (b) 5 seconds, and (c) 10 seconds.

Finding a Derivative **In Exercises 53–64, find the derivative of the function.**

53. $y = (7x + 3)^4$ **54.** $y = (x^2 - 6)^3$

55. $y = \dfrac{1}{x^2 + 4}$ **56.** $f(x) = \dfrac{1}{(5x + 1)^2}$

57. $y = 5\cos(9x + 1)$ **58.** $y = 1 - \cos 2x + 2\cos^2 x$

59. $y = \dfrac{x}{2} - \dfrac{\sin 2x}{4}$ **60.** $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$

61. $y = x(6x + 1)^5$ **62.** $f(s) = (s^2 - 1)^{5/2}(s^3 + 5)$

63. $f(x) = \dfrac{3x}{\sqrt{x^2 + 1}}$ **64.** $h(x) = \left(\dfrac{x + 5}{x^2 + 3}\right)^2$

Evaluating a Derivative **In Exercises 65–70, find and evaluate the derivative of the function at the given point.**

65. $f(x) = \sqrt{1 - x^3}$, $(-2, 3)$ **66.** $f(x) = \sqrt[3]{x^2 - 1}$, $(3, 2)$

67. $f(x) = \dfrac{4}{x^2 + 1}$, $(-1, 2)$ **68.** $f(x) = \dfrac{3x + 1}{4x - 3}$, $(4, 1)$

69. $y = \dfrac{1}{2}\csc 2x$, $\left(\dfrac{\pi}{4}, \dfrac{1}{2}\right)$

70. $y = \csc 3x + \cot 3x$, $\left(\dfrac{\pi}{6}, 1\right)$

Finding a Second Derivative **In Exercises 71–74, find the second derivative of the function.**

71. $y = (8x + 5)^3$ **72.** $y = \dfrac{1}{5x + 1}$

73. $f(x) = \cot x$ **74.** $y = \sin^2 x$

75. Refrigeration The temperature T (in degrees Fahrenheit) of food in a freezer is

$$T = \frac{700}{t^2 + 4t + 10}$$

where t is the time in hours. Find the rate of change of T with respect to t at each of the following times.

(a) $t = 1$ (b) $t = 3$ (c) $t = 5$ (d) $t = 10$

76. Harmonic Motion The displacement from equilibrium of an object in harmonic motion on the end of a spring is

$$y = \frac{1}{4}\cos 8t - \frac{1}{4}\sin 8t$$

where y is measured in feet and t is the time in seconds. Determine the position and velocity of the object when $t = \pi/4$.

Finding a Derivative **In Exercises 77–82, find dy/dx by implicit differentiation.**

77. $x^2 + y^2 = 64$ **78.** $x^2 + 4xy - y^3 = 6$

79. $x^3y - xy^3 = 4$ **80.** $\sqrt{xy} = x - 4y$

81. $x \sin y = y \cos x$ **82.** $\cos(x + y) = x$

Tangent Lines and Normal Lines **In Exercises 83 and 84, find equations for the tangent line and the normal line to the graph of the equation at the given point. (The *normal line* at a point is perpendicular to the tangent line at the point.) Use a graphing utility to graph the equation, the tangent line, and the normal line.**

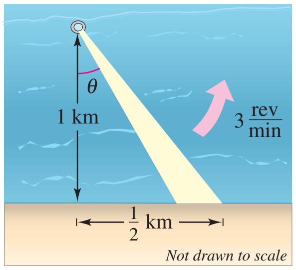
83. $x^2 + y^2 = 10$, $(3, 1)$ **84.** $x^2 - y^2 = 20$, $(6, 4)$

85. Rate of Change A point moves along the curve $y = \sqrt{x}$ in such a way that the y-value is increasing at a rate of 2 units per second. At what rate is x changing for each of the following values?

(a) $x = \frac{1}{2}$ (b) $x = 1$ (c) $x = 4$

86. Surface Area All edges of a cube are expanding at a rate of 8 centimeters per second. How fast is the surface area changing when each edge is 6.5 centimeters?

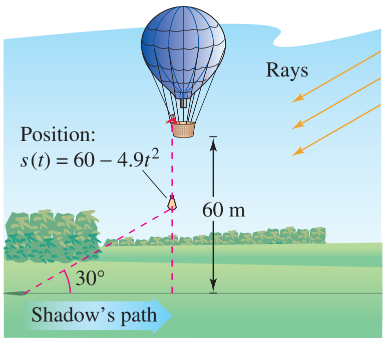
87. Linear vs. Angular Speed A rotating beacon is located 1 kilometer off a straight shoreline (see figure). The beacon rotates at a rate of 3 revolutions per minute. How fast (in kilometers per hour) does the beam of light appear to be moving to a viewer who is $\frac{1}{2}$ kilometer down the shoreline?



88. Moving Shadow A sandbag is dropped from a balloon at a height of 60 meters when the angle of elevation to the sun is 30° (see figure). The position of the sandbag is

$$s(t) = 60 - 4.9t^2.$$

Find the rate at which the shadow of the sandbag is traveling along the ground when the sandbag is at a height of 35 meters.



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